Lecture:
First Order Logic

Pros and cons of propositional logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \land P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares“
    - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Limitations of propositional logic

- ☹️ Propositional logic has limited expressive power
  - unlike natural language
  - E.g., cannot say "pits cause breezes in adjacent squares“
    - except by writing one sentence for each square

Example

- For Example

Every dog drinks water

Tommy is a dog

Brain can concludes:

Tommy drinks water

Example

- For Propositional Logic

P Every Dog drinks water
Q Tommy is a Dog
R Tommy drinks water

- But you can't go inside P & Q statement so by PL you can't conclude.

Example

- For Propositional Logic

  P Every Dog drinks water

  Q Tommy is a Dog

  R Tommy drinks water
- you can't go inside P & Q statement so by PL you can't conclude.
- But You can solve by First Order Logic

FOL Syntax

- Every FOL is divided by two parts
  - Subject
  - Predicate

- Every FOL is divided by two parts
  - Subject
  - Predicate

X is an integer.

Subject: x

Predicate: is an integer

Pinky is a cat.

Subject: Pinky

Predicate: is a cat.

FOL Syntax

- A set of predicate symbols P={P1, P2, P3, …}. We also use the symbols {P, Q, R, …|. More commonly we use words like “Man”, “Mortal”, “GreaterThan”. Each Symbol has an arity associated with it.

- A set of function symbols F={f1, f2, f3, …}. We commonly used the symbol {f,g,h, ….} or words like “Successor” and “sum”. Each function symbol has an aity that denotes the number of argument it takes.

- A set of constant symbols C={c1, c2, c3, …}. We often used symbols like “0” or “Newton” or “Kolkata” that are meaningful to us.

The three sets define a language L(P,F,C)

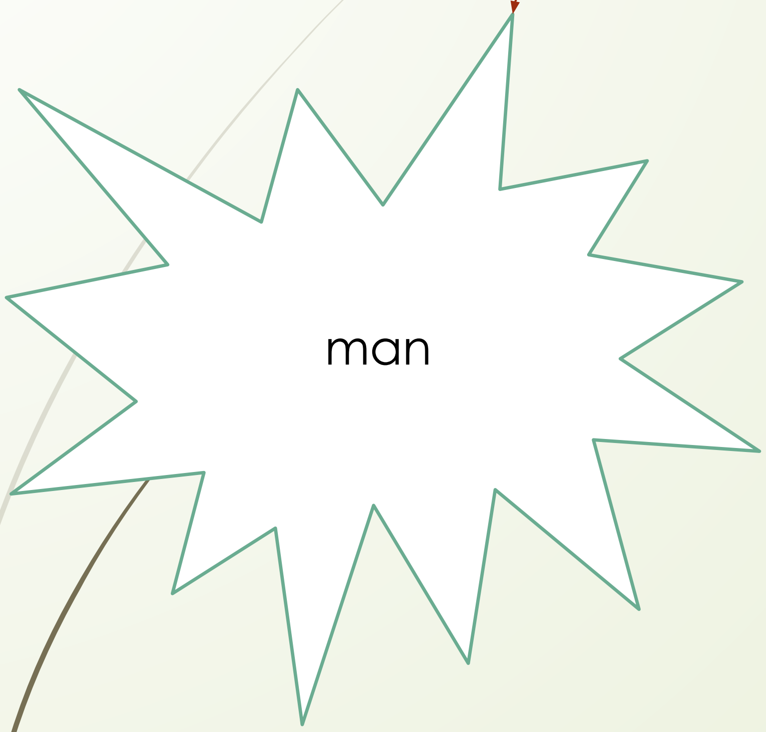
Shorthand notation

Pinky is a cat.
Subject: Pinky
Predicate: is a cat.

cat(x)= x is a cat
cat(Pinky)

Int(x) = x is an integer

"Every man drinks coffee"



x1

x2

x3

Universe of Discussion/
Domain of Discussion

x

All statement must true

$\forall x$ coffee(x)

X1 drinks coffee

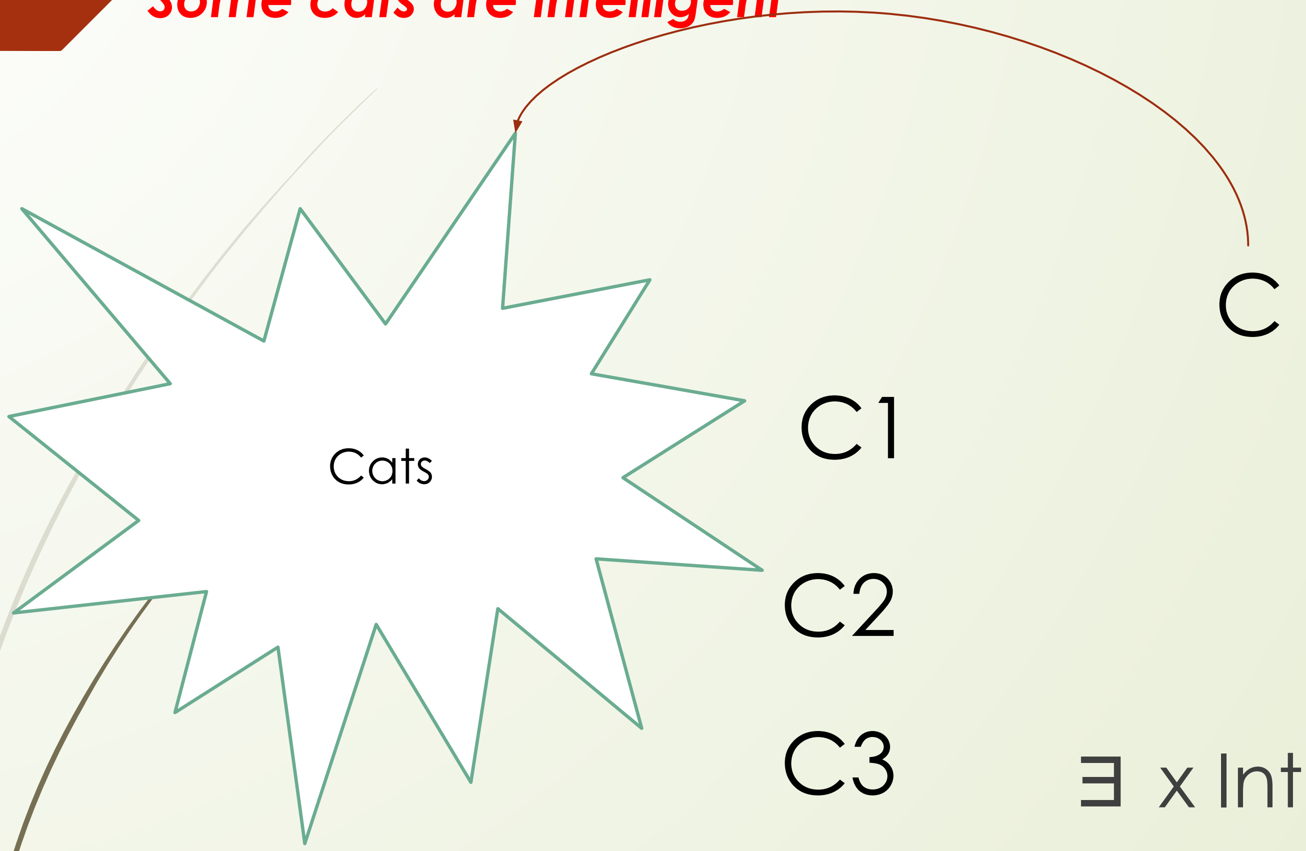
$\wedge$

X2 drinks coffee

$\wedge$

X3 drinks coffee

"Some cats are intelligent"



Universe of Discussion/
Domain of Discussion

$$\exists\, x\ \text{Intelligent}(x)$$

Some statement must true

C1 is intelligent

$\vee$

C2 is intelligent

$\vee$

C3 is intelligent

First-Order Logic

- Propositional logic assumes that the world contains facts.
- First-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Logics in General

- Ontological Commitment:
  - What exists in the world — TRUTH
  - PL : facts hold or do not hold.
  - FOL : objects with relations between them that hold or do not hold

- Epistemological Commitment:

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	degree of truth $\in [0, 1]$	known interval value

Syntax of FOL: Basic elements

- **Constant Symbols**:
  - Stand for objects
  - e.g., KingJohn, 2, UCI,...
- **Predicate Symbols:**
  - Stand for relations
  - E.g., Brother(Richard, John), greater_than(3,2)...
- **Function Symbols:**
  - Stand for functions
  - E.g., Sqrt(3), LeftLegOf(John),...

Syntax of FOL: Basic elements

- **Constants** KingJohn, 2, UCI,...
- **Predicates** Brother, >,...
- **Functions** Sqrt, LeftLegOf,...
- **Variables** x, y, a, b,...
- **Connectives** ¬, ⇒, ∧, ∨, ⇔
- **Equality** =
- **Quantifiers** ∀, ∃

Universal Quantification ∀

- ∀ means "for all"
- Allows us to make statements about all objects that have certain properties
- Can now state general rules:

$\forall x\ King(x) \rightarrow Person(x)$ "All kings are person"

$\forall x\ Person(x) \rightarrow HasHead(x)$ "Every person has a head."

Note that:

$\forall x King(x) \wedge Person(x)$ is not correct!

This would imply that all objects x are Kings and are People/Person

$\forall x King(x) \rightarrow Person(x)$ is the correct way to say

Existential Quantification ∃

- ∃ x means "there exists an x such that…." (at least one object x)
- Allows us to make statements about some object without naming it
- Examples:

  ∃ x King(x) "Some object is a king."

  ∃ x Lives_in(John, Castle(x)) "John lives in somebody's castle."

  ∃ i Integer(i) ∧ GreaterThan(i,0) "Some integer is greater than zero."

Note that:

∧ is the natural connective to use with ∃

(And → is the natural connective to use with ∀)

Nested Quantifiers

Defintion: Two quantifiers are said to be nested if one is within the scope of the other.

For example: $\forall x \exists y\, Q(x, y)$

$\exists$ is within the scope of $\forall$

Note: Anything within a scope of the quantifier can be thought of as a propositional function.

$$\forall x \boxed{\exists y\, Q(x, y)} \Rightarrow \forall x\, P(x)$$

$\exists y\, Q(x, y) \downarrow P(x)$

Different combinations of Nested Quantifiers

- $\forall x \forall y\, Q(x, y)$
- $\forall x \exists y\, Q(x, y)$
- $\exists y \forall x\, Q(x, y)$
- $\exists x \exists y\, Q(x, y)$

Order of quantifiers doesn't matter ← $\forall x \forall y\, Q(x, y)$ and $\exists x \exists y\, Q(x, y)$

$\forall x \exists y\, Q(x, y)$ and $\exists y \forall x\, Q(x, y)$ → Order of quantifiers does matter

"Some cats are intelligent"

$$\exists x[cat(x) \land I(x)]$$

“Some cats are intelligent”

- Proof that correct or wrong?

$$\exists x[cat(x) \ \square \ I(x)]$$

“*Some cats are intelligent*”

(From table: **False**)

∃x[cat(x) □ I(x)]

a1
a2
a3

Alias	Animal	Intelligent
a1	cat	No
a2	cat	No
a3	dog	Yes

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

∃a1[cat(a1) □ I(a1)] ∨ ∃a2[cat(a2) □ I(a2)] ∨ ∃a3[cat(a3) □ I(a3)]

False □True

False

False

True

True

∃x[cat(x) □ I(x)]
This is true which is contradict of the statement

“*Some cats are intelligent*”

- **Solution:**

$$\exists x[cat(x) \land I(x)]$$

Can you proof again?

“*Some cats are intelligent*”

(From table: **False**)

$\exists x[cat(x) \land I(x)]$

a1
a2
a3

Alias	Animal	Intelligent
a1	cat	No
a2	cat	No
a3	dog	Yes

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\exists a[cat(a1) \land I(a1)] \lor \exists a2[cat(a2) \land I(a2)] \lor \exists a3[cat(a3) \land I(a3)]$

False $\land$ True

False

False

False

False

$\exists x[cat(x) \land I(x)]$

This is false which is
Same as
the statement

“Every student in this class has visited Africa or America”

- Student(x): x is student in this class
- vaf(x): x has visited Africa
- vam(x): x has visited America

∀x[student(x) vaf(x) v vam(x)]

"Some prime number is even number"

- prime(x): x is prime no
- Even(x)= x is even no

$$\exists x\ [prime(x) \land even(x)]$$

"Rajiv likes Priya"

Likes(Rajiv, Priya)

“Rajiv likes Every one”

- Proof?

"Rajiv likes Everyone"

Rajiv likes x1

∧

Rajiv likes x2

∧

Rajiv likes x3

x1
x2
x3

Likes(Rajiv, x1) ∧ Likes(Rajiv, x2) ∧ Likes(Rajiv, x3)

∀xLikes(Rajiv, x)

"*Everyone likes everyone*"

- Proof?

“*Everyone likes everyone*”

Rajiv likes everyone	**∀xLikes(Rajiv, x)**
$\wedge$	
Priya likes everyone	**∀xLikes(Priya, x)**
$\wedge$	
Everyone likes Rajiv	**∀yLikes(y, Rajiv)**

…

…

…

∀y∀x[Likes(y, x)]

“Someone likes someone”

- Proof?

"*Someone likes someone*"

Rajiv likes someone $\exists y \; likes(Rajiv, y)$

…

…

$$\exists x \; \exists y \; Likes(x, y)$$

"Someone likes Everyone"

- Proof?

“Someone likes Everyone”

Rajiv likes Everyone

…

…

$\forall x$ likes(Rajiv, x)

$\exists y\ [\forall x\ Likes(y, x)]$

"*Everyone likes Someone*"

- Proof?

“Everyone likes Someone”

Rajiv likes someone **$\exists x$ Likes(Rajiv, x)]**

…

….

$\forall y$ [$\exists x$ Likes(y, x)]

“*Everyone is liked by someone*”

Rajiv is liked by someone **∃y Likes(y, Rajiv)]**

∀x ∃y Likes(y, x)

“*Someone is liked by everyone*”

- Proof?

“Someone is liked by everyone”

- Rajiv is liked by everyone

$\forall x$ Likes(x, Rajiv)]

$\exists y\ \forall x$ Likes(x, y)]

“Nobody likes everyone”

Rajiv does not like everyone

…

…

…

¬ ∀x Likes(Rajiv, x)

....

∀y [¬ ∀x Likes(y, x)]

GATE 2009

Which one of the following is the most appropriate logical formula to represent the statement? "Gold and silver ornaments are precious".

The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious

(A) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$

(B) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$

(C) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$

(D) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

Thank you!

Any Questions?